# Analysis of $Y(4660)$ and related bound states with QCD sum rules

Zhi-Gang Wang [1], Xiao-Hong Zhang
Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

**Abstract**

In this article, we take the vector charmonium-like state $Y(4660)$ as a $\psi' f_0(980)$ bound state (irrespective of the hadro-charmonium and the molecular state) tentatively, study its mass using the QCD sum rules, the numerical value $M_Y = 4.71 \pm 0.26\,\text{GeV}$ is consistent with the experimental data. Considering the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, we also study the bound states $\psi'\sigma(400-1200)$, $\Upsilon''' f_0(980)$ and $\Upsilon'''\sigma(400-1200)$ with the QCD sum rules, and make reasonable predictions for their masses.

PACS number: 12.39.Mk, 12.38.Lg
Key words: $\psi' f_0(980)$ bound state, QCD sum rules

## 1 Introduction

Two resonant structures are observed in the $\pi^+\pi^-\psi'$ invariant mass distribution in the cross section for the process $e^+e^- \to \pi^+\pi^-\psi'$ between threshold and $\sqrt{s} = 5.5$ GeV using 673 fb$^{-1}$ of data on and off the $\Upsilon(4S)$ ($\Upsilon'''$) resonance collected with the Belle detector at KEK-B, one at $4361 \pm 9 \pm 9$ MeV with a width of $74 \pm 15 \pm 10$ MeV, and another at $4664 \pm 11 \pm 5$ MeV with a width of $48 \pm 15 \pm 3$ MeV (they are denoted as $Y(4360)$ and $Y(4660)$ respectively), where the mass spectrum is parameterized by two Breit-Wigner functions [1]. The structure $Y(4660)$ is neither observed in the initial state radiation (ISR) process $e^+e^- \to \gamma_{ISR}\pi^+\pi^- J/\psi$ [2], nor in the exclusive cross processes $e^+e^- \to D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D\bar{D}\pi, J/\psi D^{(*)}\bar{D}^{(*)}$ [3, 4, 5, 6, 7].

There have been several canonical charmonium interpretations for the $Y(4660)$, such as the $5^3S_1$ state [8], the $6^3S_1$ state [9], the $5^3S_1 - 4^3D_1$ mixing state [10]. In Ref.[11], Qiao suggests that the $Y(4660)$ is a baryonium state, the radial excited state of the $\frac{1}{\sqrt{2}}(|\Lambda_c\bar{\Lambda}_c\rangle + |\Sigma^0_c\bar{\Sigma}^0_c\rangle)$. In Ref.[12], Albuquerque et al take the $Y(4660)$ as a vector $cs\bar{c}\bar{s}$ tetraquark state, and study its mass with the QCD sum rules. A critical information for understanding the structure of those charmonium-like states is wether or not the $\pi\pi$ comes from a resonance. There is some indication that only the $Y(4660)$ has a well defined intermediate state which is consistent with the scalar meson $f_0(980)$ in the $\pi\pi$ invariant mass spectra [13]. In Ref.[14], Guo et al take the $Y(4660)$ as a $\psi' f_0(980)$ bound state (molecular state) considering the nominal threshold of the $\psi' - f_0(980)$ system is about $4666 \pm 10$ MeV [15], the $Y(4660)$ decays dominantly via the decay of the scalar meson $f_0(980)$, $Y(4660) \to \psi' f_0(980) \to \psi'\pi\pi$, $\psi' K\bar{K}$, the difficulties in the canonical charmonium interpretation can be overcome. In Refs.[16, 17], Voloshin et al argue that the charmonium-like states $Y(4660)$, $Z(4430)$, $Y(4260)$, $\cdots$ may be hadro-charmonia. The relatively compact charmonium states ($J/\psi$, $\psi'$ and $\chi_{cJ}$) can be bound inside light hadronic matter, in particular inside higher resonances made from light quarks and (or) gluons. The charmonium state in such binding retains its properties essentially, the bound

[1] wangzgyiti@yahoo.com.cn.

system (hadro-charmonium, a special molecular state) decays into light mesons and the particular charmonium.

In this article, we study the mass of the $Y(4660)$ as a $\psi' f_0(980)$ bound state (irrespective of the hadro-charmonium and the molecular state) using the QCD sum rules [18, 19]. As a byproduct, we take into account the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, study the related hidden charm and hidden bottom states. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [18, 19].

The article is arranged as follows: we derive the QCD sum rules for the vector charmonium-like state $Y(4660)$ and the related bound states in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

## 2 QCD sum rules for the $Y(4660)$ and related bound states

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = i\int d^4x e^{ip\cdot x}\langle 0|T\left[J/\eta_\mu(x)J/\eta_\nu^\dagger(0)\right]|0\rangle\,, \tag{1}$$

$$\begin{aligned} J_\mu(x) &= \bar{Q}(x)\gamma_\mu Q(x)\bar{s}(x)s(x)\,, \\ \eta_\mu(x) &= \frac{1}{\sqrt{2}}\bar{Q}(x)\gamma_\mu Q(x)\left[\bar{u}(x)u(x)+\bar{d}(x)d(x)\right]\,, \end{aligned} \tag{2}$$

where the $Q$ denotes the heavy quarks $c$ and $b$. We use the currents $J_\mu(x)$ and $\eta_\mu(x)$ ($Q = c$) to interpolate the bound states $\psi' f_0(980)$ and $\psi'\sigma(400-1200)$, respectively. The $Y(4660)$ can be tentatively identified as the $\psi' f_0(980)$ bound state, while there lack experimental candidates to identify the $\psi'\sigma(400-1200)$ bound state. Considering the heavy quark symmetry, there maybe exist some hidden bottom bound states, for example, $\Upsilon f_0(980)$, $\Upsilon' f_0(980)$, $\Upsilon'' f_0(980)$, $\Upsilon''' f_0(980)$, $\Upsilon\sigma(400-1200)$, $\Upsilon'\sigma(400-1200)$, $\Upsilon''\sigma(400-1200)$, $\Upsilon'''\sigma(400-1200)$, $\cdots$, we study those possibilities with the currents $J_\mu(x)$ and $\eta_\mu(x)$ ($Q = b$), and make predictions for their masses which are fundamental parameters in describing a hadron.

The hidden charm current $\bar{c}(x)\gamma_\mu c(x)$ can interpolate the charmonia $J/\psi$, $\psi'$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, $\cdots$; while the hidden bottom current $\bar{b}(x)\gamma_\mu b(x)$ can interpolate the bottomonia $\Upsilon$, $\Upsilon'$, $\Upsilon''$, $\Upsilon'''$, $\Upsilon''''$, $\cdots$ [15]. We assume that the scalar mesons $f_0(980)$ and $\sigma(400-1200)$ are the conventional $q\bar{q}$ states, to be more precise, they have large $q\bar{q}$ components. The currents $J_\mu(x)$ and $\eta_\mu(x)$ ($Q = c$) have non-vanishing couplings with the bound states $J/\psi f_0(980)$, $\psi' f_0(980)$, $\psi'' f_0(980)$, $\cdots$ and $J/\psi\sigma(400-1200)$, $\psi'\sigma(400-1200)$, $\psi''\sigma(400-1200)$, $\cdots$, respectively. The colored objects (diquarks) in a confining potential can result in a copious spectrum, there maybe exist a series of orbital angular momentum excitations; while the colorless objects (mesons) bound by a short range potential (through meson-exchange) should have a very limited spectrum, it is relatively easy to identify the molecule type bound states. We determine the masses of the ground states by imposing the two criteria of the QCD sum rules, then compare them

with the nominal thresholds of the corresponding systems $J/\psi - f_0(980)$, $\psi' - f_0(980)$, $\cdots$. In Ref.[16], Voloshin et al argue that a formation of hadro-charmonium is favored for higher charmonium resonances $\psi'$ and $\chi_{cJ}$ as compared to the lowest states $J/\psi$ and $\eta_c$.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_\mu(x)$ and $\eta_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [18, 19]. After isolating the ground state contributions from the pole terms of the $Y$ and $Z$, we get the following result,

$$\Pi_{\mu\nu}(p) = \frac{\lambda_Y^2}{M_Y^2 - p^2}\left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right] + \frac{\lambda_Z^2}{M_Z^2 - p^2} p_\mu p_\nu + \cdots \, , \tag{3}$$

where the pole residues (or coupling) $\lambda_Y$ and $\lambda_Z$ are defined by

$$\begin{aligned} \lambda_Y \epsilon_\mu &= \langle 0|J/\eta_\mu(0)|Y(p)\rangle \, , \\ \lambda_Z p_\mu &= \langle 0|J/\eta_\mu(0)|Z(p)\rangle \, , \end{aligned} \tag{4}$$

and the $\epsilon_\mu$ is the polarization vector. In Eq.(3), we show the contribution from the scalar bound state $Z$ explicitly, because the vector currents $J_\mu(x)$ and $\eta_\mu(x)$ are by no means conserved.

After performing the standard procedure of the QCD sum rules, we obtain two sum rules for the $c\bar{c}s\bar{s}$ and $b\bar{b}s\bar{s}$ channels respectively (In the isospin limit, the interpolating currents result in two distinct expressions for the correlation functions $\Pi_{\mu\nu}(p)$, which are characterized by the number of the $s$ quark they contain, thereafter will use the quark constituents to denote the corresponding quantities.):

$$\lambda_Y^2 e^{-\frac{M_Y^2}{M^2}} = \int_\Delta^{s_0} ds \rho(s) e^{-\frac{s}{M^2}} \, , \tag{5}$$

$$\rho(s) = \rho_0(s) + \rho_{\langle \bar{s}s\rangle}(s) + \left[\rho^A_{\langle GG\rangle}(s) + \rho^B_{\langle GG\rangle}(s)\right] \langle \frac{\alpha_s GG}{\pi}\rangle + \rho_{\langle \bar{s}s\rangle^2}(s) \, . \tag{6}$$

The explicit expressions of the spectral densities $\rho_0(s)$, $\rho_{\langle \bar{s}s\rangle}(s)$, $\rho^A_{\langle GG\rangle}(s)$, $\rho^B_{\langle GG\rangle}(s)$ and $\rho_{\langle \bar{s}s\rangle^2}(s)$ are presented in the appendix. The $s_0$ is the continuum threshold parameter and the $M^2$ is the Borel parameter; $\alpha_f = \frac{1+\sqrt{1-4m_Q^2/s}}{2}$, $\alpha_i = \frac{1-\sqrt{1-4m_Q^2/s}}{2}$, $\beta_i = \frac{\alpha m_Q^2}{\alpha s - m_Q^2}$, $\widetilde{m}_Q^2 = \frac{(\alpha+\beta)m_Q^2}{\alpha\beta}$, $\widetilde{\widetilde{m}}_Q^2 = \frac{m_Q^2}{\alpha(1-\alpha)}$, and $\Delta = 4(m_Q + m_s)^2$. We can obtain two sum rules for the $c\bar{c}q\bar{q}$ and $b\bar{b}q\bar{q}$ channels with a simple replacement $m_s \to 0$, $\langle \bar{s}s\rangle \to \langle \bar{q}q\rangle$ and $\langle \bar{s}g_s\sigma Gs\rangle \to \langle \bar{q}g_s\sigma Gq\rangle$.

We carry out the operator product expansion (OPE) to the vacuum condensates adding up to dimension-10. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in the large $N_c$ limit. Moreover, we neglect the terms proportional to the $m_u$ and $m_d$, their contributions are of minor importance.

Differentiate the Eq.(5) with respect to $\frac{1}{M^2}$, then eliminate the pole residue $\lambda_Y$, we can obtain the sum rule for the mass of the bound state $Y$,

$$M_Y^2 = \frac{\int_\Delta^{s_0} ds \frac{d}{d(-1/M^2)} \rho(s) e^{-\frac{s}{M^2}}}{\int_\Delta^{s_0} ds \rho(s) e^{-\frac{s}{M^2}}} \, . \tag{7}$$

# 3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle = (0.8 \pm 0.2)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8 \pm 0.2)\,\text{GeV}^2$, $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\text{GeV})^4$, $m_s = (0.14 \pm 0.01)\,\text{GeV}$, $m_c = (1.35 \pm 0.10)\,\text{GeV}$ and $m_b = (4.8 \pm 0.1)\,\text{GeV}$ at the energy scale $\mu = 1\,\text{GeV}$ [18, 19, 20].

In the conventional QCD sum rules [18, 19], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. We impose the two criteria on the charmonium-like states $Y$ to choose the Borel parameter $M^2$ and threshold parameter $s_0$. The light tetraquark states cannot satisfy the two criteria, although it is not an indication non-existence of the light tetraquark states (For detailed discussions about this subject, one can consult Refs.[21, 22]).

We take the vector charmonium-like state $Y(4660)$ as the $\psi' f_0(980)$ bound state tentatively, and take the threshold parameter as $s^0_{s\bar{s}} = (4.66 + 0.5)^2\,\text{GeV}^2 \approx 27\,\text{GeV}^2$ to take into account possible contribution from the ground state, where we choose the energy gap between the ground state and the first radial excited state to be 0.5 GeV. Taking into account the $SU(3)$ symmetry of the light flavor quarks, we expect the threshold parameter $s^0_{q\bar{q}}$ (for the bound state $\psi'\sigma(400-1200)$) is slightly smaller than the $s^0_{s\bar{s}}$. Furthermore, we take into account the mass difference between the $c$ and $b$ quarks, the threshold parameters in the hidden bottom channels are tentatively taken as $s^0_{q\bar{q}} = 144\,\text{GeV}^2$ and $s^0_{s\bar{s}} = 145\,\text{GeV}^2$.

In this article, we take it for granted that the energy gap between the ground state and the first radial excited state is about 0.5 GeV, and use this value as a guide to determine the threshold parameter $s_0$ with the QCD sum rules.

The contributions from the high dimension vacuum condensates in the operator product expansion are shown in Figs.1-2, where (and thereafter) we use the $\langle\bar{q}q\rangle$ to denote the quark condensates $\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$ and the $\langle\bar{q}g_s\sigma Gq\rangle$ to denote the mixed condensates $\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle$. From the figures, we can see that the contributions from the high dimension condensates change quickly with variation of the Borel parameter at the values $M^2 \leq 2.8\,\text{GeV}^2$ and $M^2 \leq 7.5\,\text{GeV}^2$ for the hidden charm and hidden bottom channels respectively, such an unstable behavior cannot lead to stable sum rules, our numerical results confirm this conjecture, see Fig.4.

At the values $M^2 \geq 2.8\,\text{GeV}^2$ and $s_0 \geq 26\,\text{GeV}^2$, the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term are less than (or equal) 18.5% for the $c\bar{c}s\bar{s}$ channel, the corresponding contributions are less than (or equal) 36.5% for the $c\bar{c}q\bar{q}$ channel; the contributions from the vacuum condensate of the highest dimension $\langle\bar{q}g_s\sigma Gq\rangle^2$ are less than 5% for all the hidden charm channels, we expect the operator product expansion is convergent in the hidden charm channels. At the values $M^2 \geq 7.6\,\text{GeV}^2$ (In Figs.2-4, the vertical line corresponds to the value $M^2 = 7.6\,\text{GeV}^2$ in the hidden bottom channels.) and $s_0 \geq 148\,\text{GeV}^2$, the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term are less than 7% for the $b\bar{b}s\bar{s}$ channel, the corresponding contributions are less than (or equal) 18% for the $b\bar{b}q\bar{q}$ channel; the contributions from the vacuum condensate of the highest dimension $\langle\bar{q}g_s\sigma Gq\rangle^2$ are less than (or equal) 7% for all the hidden bottom channels, we expect the operator product expansion is convergent in the hidden bottom channels.

The contributions from the gluon condensate $\langle\frac{\alpha_s GG}{\pi}\rangle$ are rather large, while the contri-

butions from the high dimension condensates $\langle\frac{\alpha_s GG}{\pi}\rangle\left[\langle\bar{q}q\rangle+\langle\bar{q}g_s\sigma Gq\rangle+\langle\bar{q}q\rangle^2\right]$ are small enough, the total contributions involving the gluon condensate are less than (or equal) 30% (22%) for the $c\bar{c}s\bar{s}$ ($c\bar{c}q\bar{q}$) channel at the values $M^2\geq 2.8\,\mathrm{GeV}^2$ and $s_0\geq 26\,\mathrm{GeV}^2$; while the contributions are less than 21% (17%) for the $b\bar{b}s\bar{s}$ ($b\bar{b}q\bar{q}$) channel at the values $M^2\geq 7.6\,\mathrm{GeV}^2$ and $s_0\geq 148\,\mathrm{GeV}^2$. In the QCD sum rules for the tetraquark states (irrespective of the molecule type and the diquark-antidiquark type), the contributions from the gluon condensate are suppressed by large denominators and would not play any significant roles for the light tetraquark states [23, 24], the heavy tetraquark state [21] and the heavy molecular state [25]; the present sum rules seem rather exotic. If we take a simple replacement $\bar{s}(x)s(x)\to\langle\bar{s}s\rangle$ and $\left[\bar{u}(x)u(x)+\bar{d}(x)d(x)\right]\to 2\langle\bar{q}q\rangle$ in the interpolating currents $J_\mu(x)$ and $\eta_\mu(x)$, we can obtain the standard vector heavy quark current $Q(x)\gamma_\mu Q(x)$, where the gluon condensate plays an important rule in the QCD sum rules [18].

In calculation, we observe that the dominant contributions come from the perturbative term and the $\langle\bar{q}q\rangle+\langle\bar{q}g_s\sigma Gq\rangle$ term at the values $M^2\geq 2.8\,\mathrm{GeV}^2$ and $s_0\geq 26\,\mathrm{GeV}^2$ for the hidden charm channels and at the values $M^2\geq 7.6\,\mathrm{GeV}^2$ and $s_0\geq 148\,\mathrm{GeV}^2$ for the hidden bottom channels, the operator product expansion is convergent.

In this article, we take the uniform Borel parameter $M^2_{min}$, i.e. $M^2_{min}\geq 2.8\,\mathrm{GeV}^2$ and $M^2_{min}\geq 7.6\,\mathrm{GeV}^2$ for the hidden charm and hidden bottom channels, respectively.

In Fig.3, we show the contributions from the pole terms with variation of the Borel parameters $M^2$ and the threshold parameters $s_0$. If the pole dominance criterion is satisfied, the threshold parameter $s_0$ increases with the Borel parameter $M^2$ monotonously. From Fig.3-A, we can see that the pole dominance criterion cannot be satisfied at the values $s_0\leq 25\,\mathrm{GeV}^2$ and $M^2\geq 2.8\,\mathrm{GeV}^2$ in the $c\bar{c}s\bar{s}$ channel, the threshold parameter $s_0$ has to be pushed to larger value.

The pole contributions are larger than 45% at the values $M^2\leq 3.2\,\mathrm{GeV}^2$ and $s_0\geq 25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$ for the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$ channels respectively; and larger than 50% at the values $M^2\leq 8.2\,\mathrm{GeV}^2$, $s_0\geq 146\,\mathrm{GeV}^2$, $148\,\mathrm{GeV}^2$ for the $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels respectively. Again we take the uniform Borel parameter $M^2_{max}$, i.e. $M^2_{max}\leq 3.2\,\mathrm{GeV}^2$ and $M^2_{max}\leq 8.2\,\mathrm{GeV}^2$ for the hidden charm and hidden bottom channels, respectively.

If we take uniform pole contributions, the interpolating current with more $s$ quarks requires larger threshold parameter due to the $SU(3)$ breaking effects, see Fig.3. The threshold parameters in the $c\bar{c}q\bar{q}$ and $b\bar{b}q\bar{q}$ channels are slightly smaller than the ones in the $c\bar{c}s\bar{s}$ and $b\bar{b}s\bar{s}$ channels respectively. In this article, the threshold parameters are taken as $s_0=(26\pm1)\,\mathrm{GeV}^2$, $(27\pm1)\,\mathrm{GeV}^2$, $(148\pm2)\,\mathrm{GeV}^2$ and $(150\pm2)\,\mathrm{GeV}^2$ for the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels, respectively; the Borel parameters are taken as $M^2=(2.8-3.2)\,\mathrm{GeV}^2$ and $(7.6-8.2)\,\mathrm{GeV}^2$ for the hidden charm and hidden bottom channels, respectively. In those regions, the pole contributions are about $(45-69)\%$, $(46-69)\%$, $(50-66)\%$ and $(51-67)\%$ for the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively; the two criteria of the QCD sum rules are fully satisfied [18, 19]. Naively, we expect the bound state with the scalar meson $f_0(980)$ will have larger mass than the corresponding one with the scalar meson $\sigma(400-1200)$, our numerical calculations confirm this conjecture, see Fig.4. Although smaller threshold parameters lead to slower convergent behavior in the operator product expansion, the two criteria of the QCD sum rules are still satisfied, one can consult Figs.1-2.

The Borel windows $M^2_{max}-M^2_{min}$ change with variations of the threshold parameters

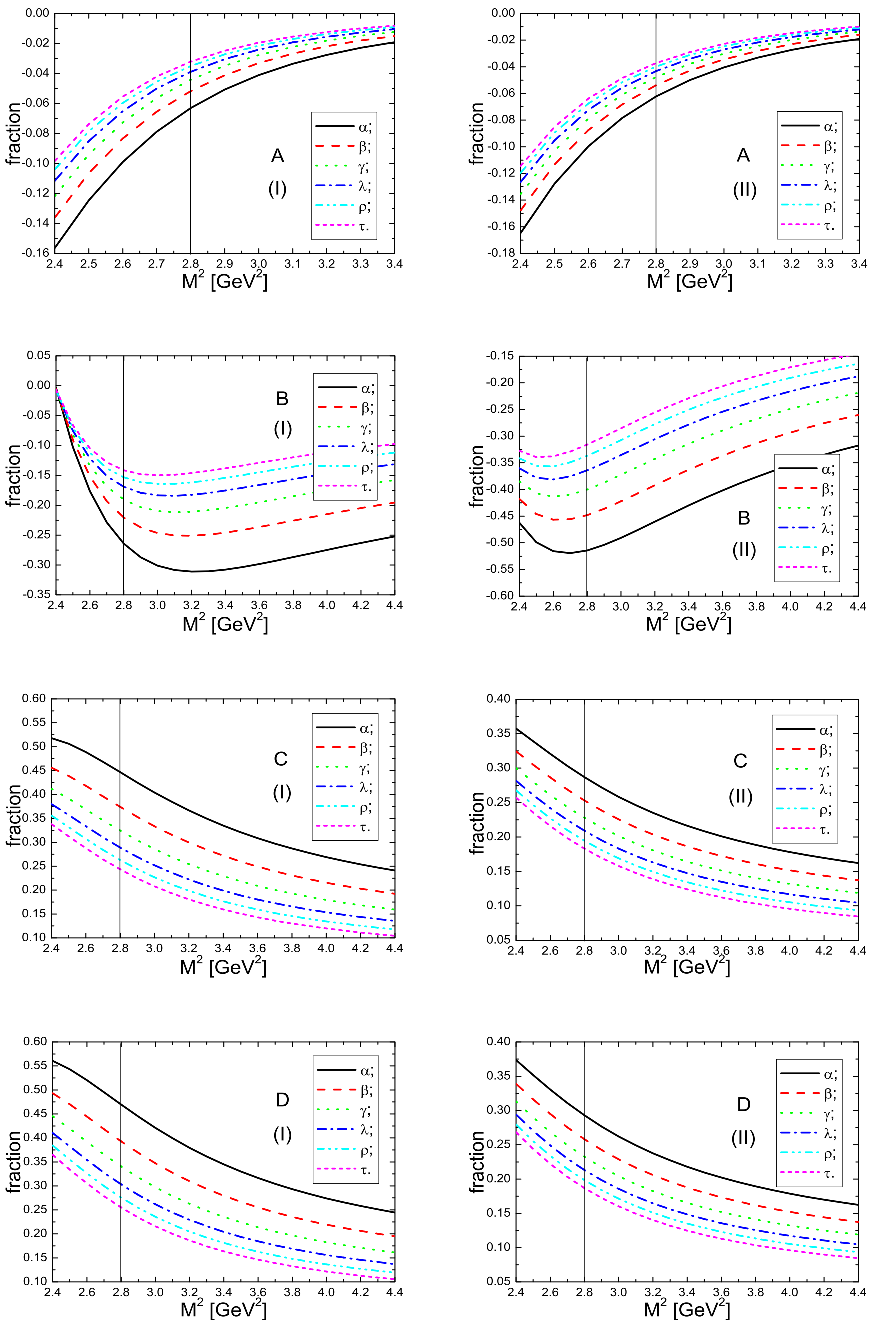


Figure 1: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion. The $A$, $B$, $C$ and $D$ correspond to the contributions from the $\langle\bar{q}g_s\sigma Gq\rangle^2$ term, the $\langle\bar{q}q\rangle^2$ $+\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term, the $\langle\frac{\alpha_sGG}{\pi}\rangle$ term and the $\langle\frac{\alpha_sGG}{\pi}\rangle+\langle\frac{\alpha_sGG}{\pi}\rangle\left[\langle\bar{q}q\rangle+\langle\bar{q}g_s\sigma Gq\rangle+\langle\bar{q}q\rangle^2\right]$ term, respectively. The (I) and (II) denote the $c\bar{c}s\bar{s}$ and $c\bar{c}q\bar{q}$ channels, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 23\,\mathrm{GeV}^2$, $24\,\mathrm{GeV}^2$, $25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$, $27\,\mathrm{GeV}^2$ and $28\,\mathrm{GeV}^2$, respectively.

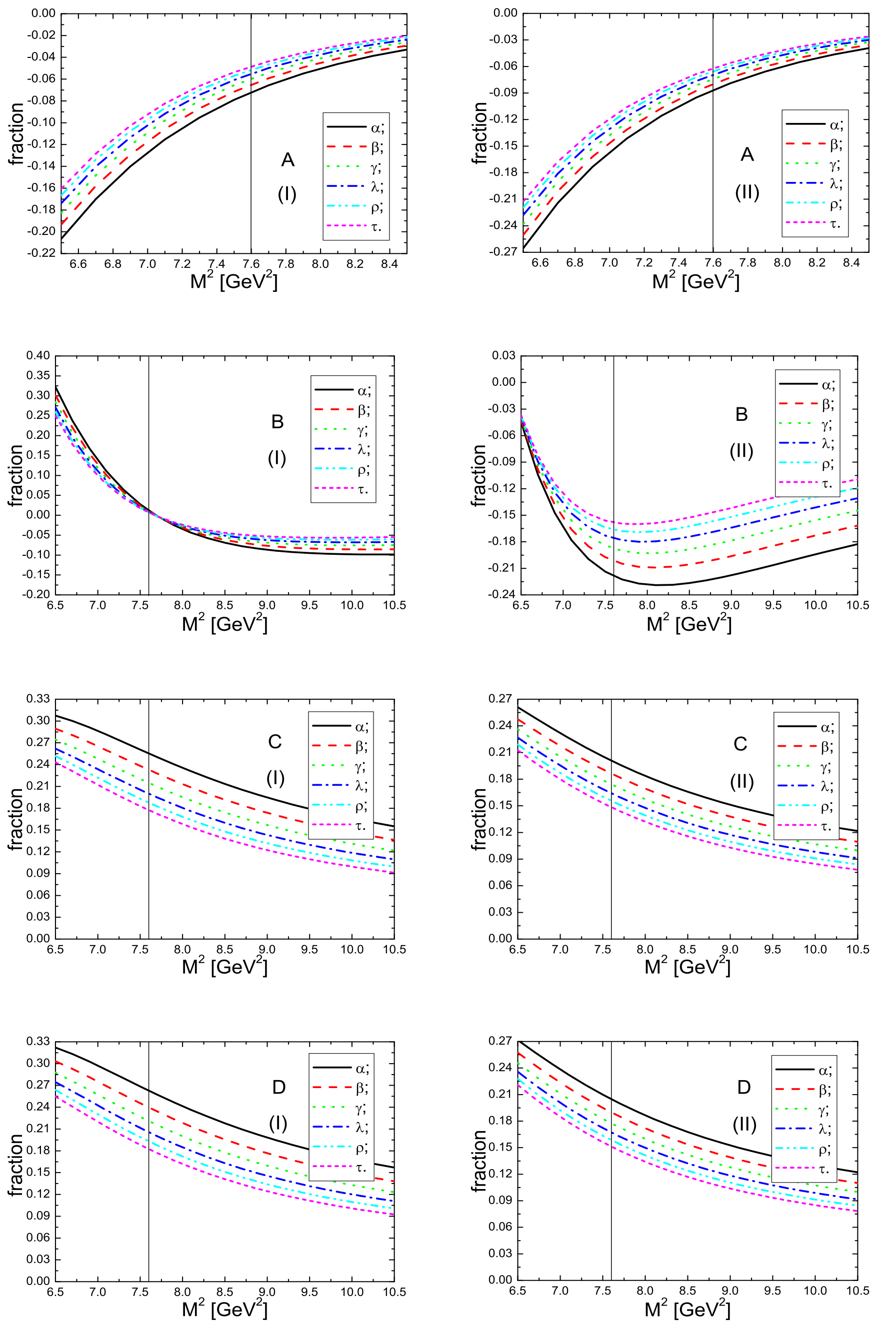


Figure 2: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion. The $A$, $B$, $C$ and $D$ correspond to the contributions from the $\langle\bar{q}g_s\sigma Gq\rangle^2$ term, the $\langle\bar{q}q\rangle^2$ $+\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term, the $\langle\frac{\alpha_sGG}{\pi}\rangle$ term and the $\langle\frac{\alpha_sGG}{\pi}\rangle+\langle\frac{\alpha_sGG}{\pi}\rangle\left[\langle\bar{q}q\rangle+\langle\bar{q}g_s\sigma Gq\rangle+\langle\bar{q}q\rangle^2\right]$ term, respectively. The (I) and (II) denote the $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 142\,\mathrm{GeV}^2$, $144\,\mathrm{GeV}^2$, $146\,\mathrm{GeV}^2$, $148\,\mathrm{GeV}^2$, $150\,\mathrm{GeV}^2$ and $152\,\mathrm{GeV}^2$, respectively.

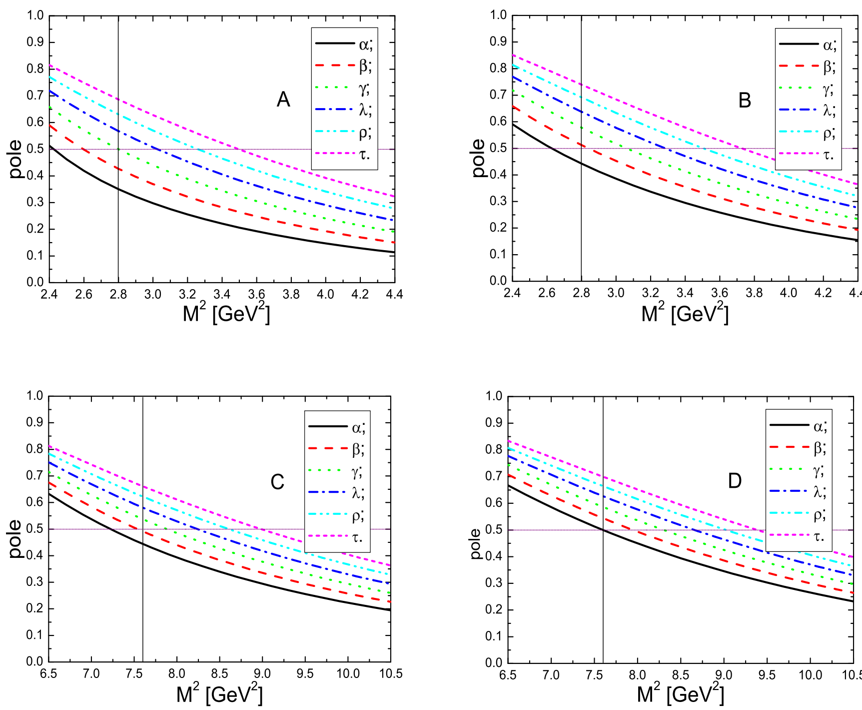


Figure 3: The contributions from the pole terms with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. In the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 23\,\mathrm{GeV}^2$, $24\,\mathrm{GeV}^2$, $25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$, $27\,\mathrm{GeV}^2$ and $28\,\mathrm{GeV}^2$, respectively ; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 142\,\mathrm{GeV}^2$, $144\,\mathrm{GeV}^2$, $146\,\mathrm{GeV}^2$, $148\,\mathrm{GeV}^2$, $150\,\mathrm{GeV}^2$ and $152\,\mathrm{GeV}^2$, respectively.

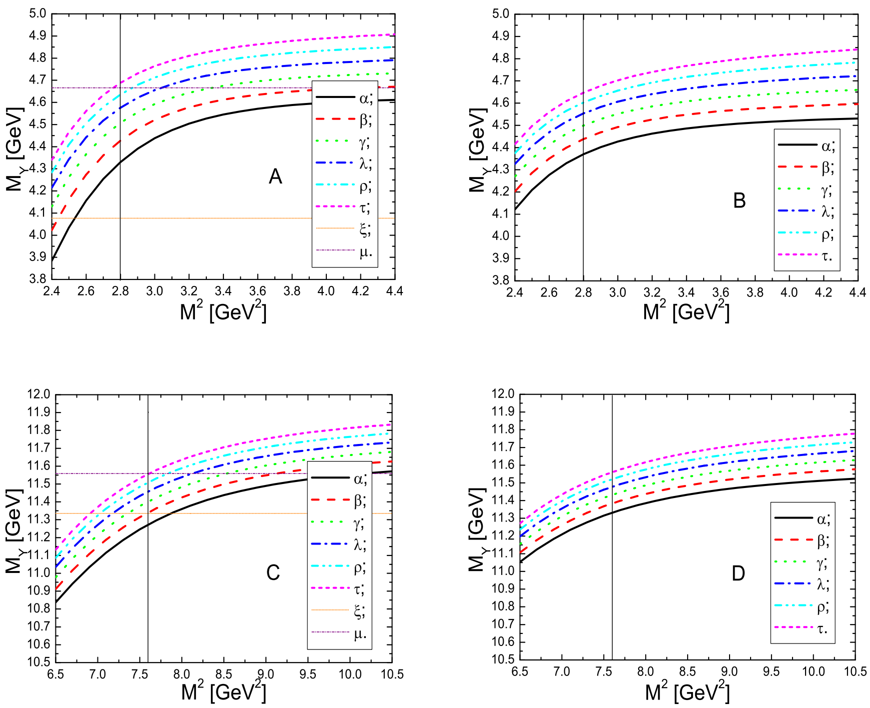


Figure 4: The masses of the vector bound states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, and $b\bar{b}q\bar{q}$ channels, respectively. In the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 23\,\mathrm{GeV}^2$, $24\,\mathrm{GeV}^2$, $25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$, $27\,\mathrm{GeV}^2$ and $28\,\mathrm{GeV}^2$, respectively ; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 142\,\mathrm{GeV}^2$, $144\,\mathrm{GeV}^2$, $146\,\mathrm{GeV}^2$, $148\,\mathrm{GeV}^2$, $150\,\mathrm{GeV}^2$ and $152\,\mathrm{GeV}^2$, respectively. The $\xi$ and $\mu$ denote the $J/\psi - f_0(980)$ and $\psi' - f_0(980)$ thresholds respectively in the $c\bar{c}s\bar{s}$ channel, while in the $b\bar{b}s\bar{s}$ channel they correspond to $\Upsilon'' - f_0(980)$ and $\Upsilon''' - f_0(980)$ thresholds respectively.

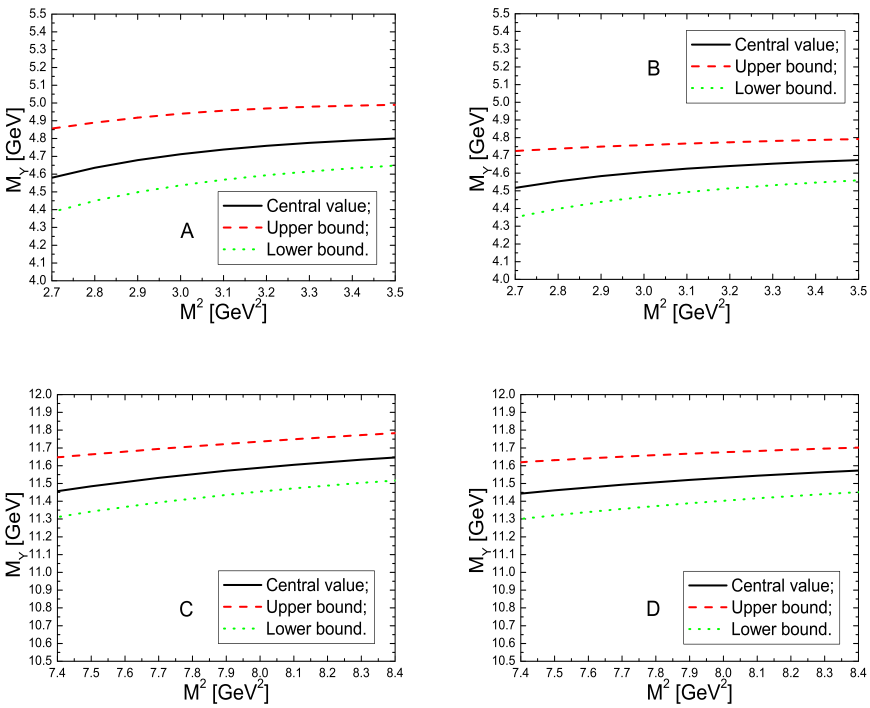


Figure 5: The masses of the vector bound states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, and $b\bar{b}q\bar{q}$ channels, respectively.

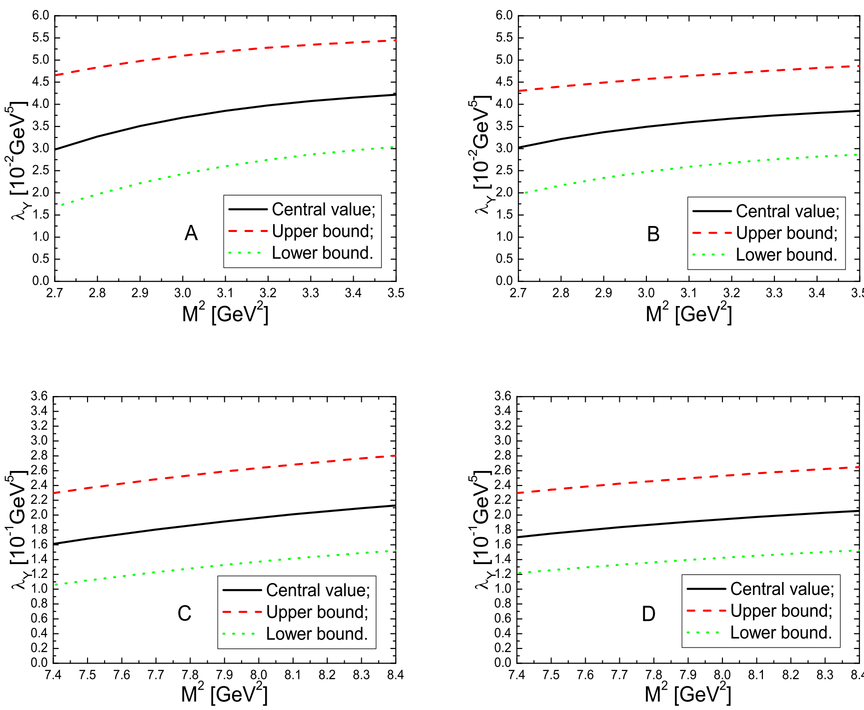


Figure 6: The pole residues of the vector bound states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, and $b\bar{b}q\bar{q}$ channels, respectively.

$s_0$, see Fig.3. In this article, the Borel windows are taken as $0.4\,\mathrm{GeV}^2$ and $0.6\,\mathrm{GeV}^2$ for the hidden charm and hidden bottom channels respectively, they are small enough. Furthermore, we take uniform Borel windows and smear the dependence on the threshold parameters $s_0$. If we take larger threshold parameters, the Borel windows are larger and the resulting masses are larger, see Fig.4. In this article, we intend calculate the possibly lowest masses which are supposed to be the ground state masses by imposing the two criteria of the QCD sum rules.

In Fig.4, we plot the bound state masses $M_Y$ with variation of the Borel parameters and the threshold parameters. The hidden charm current $\bar{c}(x)\gamma_\mu c(x)$ can interpolate the charmonia $J/\psi$, $\psi'$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, $\cdots$; while the hidden bottom current $\bar{b}(x)\gamma_\mu b(x)$ can interpolate the bottomonia $\Upsilon$, $\Upsilon'$, $\Upsilon''$, $\Upsilon'''$, $\Upsilon''''$, $\cdots$ [15]. The currents $J_\mu(x)$ have non-vanishing couplings with the bound states $J/\psi f_0(980)$, $\psi' f_0(980)$, $\psi'' f_0(980)$, $\cdots$ and $\Upsilon f_0(980)$, $\Upsilon' f_0(980)$, $\Upsilon'' f_0(980)$, $\Upsilon''' f_0(980)$, $\cdots$, respectively. From Figs.3-A,3-C,4-A,4-C, we can see that the QCD sum rules support existence of the $\psi' f_0(980)$ and $\Upsilon''' f_0(980)$ bound states, the nominal thresholds of the $J/\psi - f_0(980)$ and $\Upsilon'' - f_0(980)$ systems are too low, and we cannot reproduce the $J/\psi f_0(980)$ and $\Upsilon'' f_0(980)$ bound states. Our numerical results support the conjecture of Voloshin et al, i.e. a formation of hadro-charmonium is favored for higher charmonium resonances $\psi'$ and $\chi_{cJ}$ as compared to the lowest states $J/\psi$ and $\eta_c$ [16].

In this article, we intend prove that the $\psi' f_0(980)$ and $\Upsilon''' f_0(980)$ bound states can be reproduced by the QCD sum rules, the charmonium-like state $Y(4660)$ has the possibility to be a $\psi' f_0(980)$ bound state.

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole resides of the vector bound states $Y$, which are shown in Figs.5-6 and Tables 1-2. In this article, we calculate the uncertainties $\delta$ with the formula

$$\delta = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 |_{x_i=\bar{x}_i} (x_i - \bar{x}_i)^2}\,, \tag{8}$$

where the $f$ denote the hadron mass $M_Y$ and the pole residue $\lambda_Y$, the $x_i$ denote the input QCD parameters $m_c$, $m_b$, $\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$, $\cdots$. As the partial derivatives $\frac{\partial f}{\partial x_i}$ are difficult to carry out analytically, we take the approximation $\left(\frac{\partial f}{\partial x_i}\right)^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i)]^2$ in the numerical calculations.

From Tables 1-2, we can see that the uncertainties of the masses $M_Y$ are rather small (about 5% in the hidden charm channels and 2% in the hidden bottom channels), while the uncertainties of the pole residues $\lambda_Y$ are rather large (about $(30-50)\%$). The uncertainties of the input parameters ($\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle$, $m_s$, $m_c$ and $m_b$) vary in the range $(2-25)\%$, the uncertainties of the pole residues $\lambda_Y$ are reasonable. We obtain the squared masses $M_Y^2$ through a fraction, see Eq.(7), the uncertainties in the numerator and denominator which origin from a given input parameter (for example, $\langle\bar{s}s\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle$) cancel out with each other, and result in small net uncertainty.

In table 1, we also present the nominal thresholds of the $\psi' - f_0(980)$, $\psi' - \sigma(400-1200)$, $\Upsilon''' - f_0(980)$ and $\Upsilon''' - \sigma(400-1200)$ systems. From the table, we can see that the $Y(4660)$ can be tentatively identified as the $\psi' f_0(980)$ bound state. The predicted mass of the $\psi'\sigma(400-1200)$ bound state is about $(4.59 \pm 0.19)\mathrm{GeV}$, while the nominal threshold

| bound states | $M_Y$ (GeV) | $M_{\psi'/\Upsilon'''}+M_{f_0/\sigma}$ (GeV) | $M_Y$ (GeV)$*$ |
|---|---|---|---|
| $c\bar{c}s\bar{s}$ | $4.71\pm0.26$ | 4.666 | 4.63 |
| $c\bar{c}q\bar{q}$ | $4.59\pm0.19$ | $4.086-4.886$ | 4.56 |
| $b\bar{b}s\bar{s}$ | $11.57\pm0.20$ | 11.559 | 11.56 |
| $b\bar{b}q\bar{q}$ | $11.52\pm0.18$ | $10.979-11.779$ | 11.51 |

Table 1: The masses of the bound states, we use the star $*$ to denote the central values from the sum rules where the perturbative contributions are multiplied by a factor 2.

| bound states | $\lambda_Y$ $(10^{-2}\mathrm{GeV}^5)$ | $\lambda_Y$ $(10^{-2}\mathrm{GeV}^5)*$ |
|---|---|---|
| $c\bar{c}s\bar{s}$ | $3.70^{+1.58}_{-1.74}$ | 5.23 |
| $c\bar{c}q\bar{q}$ | $3.49^{+1.21}_{-1.32}$ | 4.84 |
| $b\bar{b}s\bar{s}$ | $19.2\pm8.2$ | 28.6 |
| $b\bar{b}q\bar{q}$ | $19.2\pm6.7$ | 27.3 |

Table 2: The pole residues of the bound states, we use the star $*$ to denote the central values from the sum rules where the perturbative contributions are multiplied by a factor 2.

of the $\psi'-\sigma(400-1200)$ system is about $(4.086-4.886)\,\mathrm{GeV}$. There maybe exist such a bound state. The $\psi'\sigma(400-1200)$ bound state can be produced in the initial state radiation process $e^+e^-\to\gamma_{ISR}\pi^+\pi^-\psi'$ or in the exclusive decays of the $B$ meson through $b\to c\bar{c}q$ at the quark level. There still lack experimental candidates to identify the $\psi'\sigma(400-1200)$ bound state, such a bound state is difficult to observe due to the broad width of the scalar meson $\sigma(400-1200)$.

In the $b\bar{b}s\bar{s}$ channel, the numerical result $M_Y=11.57\pm0.20\,\mathrm{GeV}$ indicates that there maybe exist a $\Upsilon'''f_0(980)$ bound state, which is consistent with the nominal threshold $M_{\Upsilon'''}+M_{f_0}=11.559\,\mathrm{GeV}$, while the nominal thresholds $M_{\Upsilon}+M_{f_0}=10.44\,\mathrm{GeV}$, $M_{\Upsilon'}+M_{f_0}=11.00\,\mathrm{GeV}$, $M_{\Upsilon''}+M_{f_0}=11.335\,\mathrm{GeV}$ are too low. The scalar meson $\sigma(400-1200)$ is rather broad with the Breit-Wigner mass formula $(400-1200)-i(250-500)$ [15]. Considering the $SU(3)$ symmetry of the light flavor quarks, we can obtain the conclusion tentatively that there maybe exist the $\psi'\sigma(400-1200)$ and $\Upsilon'''\sigma(400-1200)$ bound states which lie in the regions $(4.086-4.886)\,\mathrm{GeV}$ and $(10.979-11.779)\,\mathrm{GeV}$, respectively. As the energy gaps between the $\Upsilon$'s are rather small and the scalar meson $\sigma(400-1200)$ is broad enough, there maybe exist the $\Upsilon\sigma(400-1200)$, $\Upsilon'\sigma(400-1200)$ and $\Upsilon''\sigma(400-1200)$ bound states. We cannot draw decisive conclusion with the QCD sum rules alone.

At the energy scale $\mu=1\,\mathrm{GeV}$, $\frac{\alpha_s}{\pi}\approx0.19$ [26], if the perturbative $\mathcal{O}(\alpha_s)$ corrections to the perturbative term are companied with large numerical factors, $1+\xi(s,m_Q)\frac{\alpha_s}{\pi}$, for example, $\xi(s,m_Q)>\frac{\pi}{\alpha_s}\approx5$, the contributions may be large. We can make a crude estimation by multiplying the perturbative term with a numerical factor, say $1+\xi(s,m_Q)\frac{\alpha_s}{\pi}=2$, the masses $M_Y$ decrease slightly while the pole residues $\lambda_Y$ increase remarkably, see Tables 1-2. The main contribution comes from the perturbative term, the large corrections in the numerator and denominator cancel out with each other (see Eq.(7)). In fact, the

$\xi(s, m_Q)$ are complicated functions of the energy $s$ and the mass $m_Q$, such a crude estimation maybe underestimate the $\mathcal{O}(\alpha_s)$ corrections, the uncertainties originate from the $\mathcal{O}(\alpha_s)$ corrections maybe larger.

The charmonia $J/\psi$, $\psi'$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, $\cdots$ and the bottomonia $\Upsilon$, $\Upsilon'$, $\Upsilon''$, $\Upsilon'''$, $\Upsilon''''$, $\cdots$ also have Fock states with additional $q\bar{q}$ components beside the $Q\bar{Q}$ components. The currents $J_\mu(x)$ and $\eta_\mu(x)$ may have non-vanishing couplings with the charmonia and bottomonia, those couplings are supposed to be small, as the main Fock states of the charmonia and bottomonia are the $Q\bar{Q}$ components, and the charmonia and bottomonia have much smaller masses than the corresponding molecular states $Y$.

In this article, we take the assumption that the scalar mesons $f_0(980)$ and $\sigma(400-1200)$ are the conventional $q\bar{q}$ mesons, or more precise, they have large $q\bar{q}$ components. There are hot controversies about their nature, for example, the conventional $q\bar{q}$ states (strongly affected by the nearby thresholds), the tetraquark states, the molecular states [27, 28]. In Ref.[29], we take the scalar mesons $a_0(980)$ and $f_0(980)$ as the conventional $q\bar{q}$ mesons, study the strong couplings to the nearby thresholds, and observe that the strong couplings are rather large. Then we draw the conclusion that the $a_0(980)$ and $f_0(980)$ may have a small $q\bar{q}$ kernel of the typical $q\bar{q}$ meson size, strong coupling to the nearby $\bar{K}K$ threshold may result in some tetraquark components (irrespective of a nucleon-like bound state and a deuteron-like bound state) [29]. The decay $f_0(980)/\sigma(400-1200) \to \pi\pi, K\bar{K}$ can occur through the tetraquark quark components naturally.

The LHCb is a dedicated $b$ and $c$-physics precision experiment at the LHC (large hadron collider). The LHC will be the world's most copious source of the $b$ hadrons, and a complete spectrum of the $b$ hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14\,\text{TeV}$, the $b\bar{b}$ cross section is expected to be $\sim 500\mu b$ producing $10^{12}$ $b\bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ [30]. The bound states $\Upsilon''' f_0(980)$ and $\Upsilon'''\sigma(400-1200)$ predicted in the present work may be observed at the LHCb, if they exist indeed. We can search for those bound states in the $\Upsilon\pi\pi$, $\Upsilon'\pi\pi$, $\Upsilon''\pi\pi$, $\Upsilon'''\pi\pi$, $\Upsilon K\bar{K}$, $\Upsilon' K\bar{K}$, $\Upsilon'' K\bar{K}$, $\Upsilon''' K\bar{K}$, $\cdots$ invariant mass distributions.

## 4 Conclusion

In this article, we take the the vector charmonium-like state $Y(4660)$ as the $\psi' f_0(980)$ bound state (irrespective of the hadro-charmonium and the molecular state) tentatively, study its mass using the QCD sum rules, the numerical result $M_Y = 4.71 \pm 0.26\,\text{GeV}$ is consistent with the experimental data $4664 \pm 11 \pm 5$ MeV. Considering the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, we also study the bound states $\psi'\sigma(400-1200)$, $\Upsilon''' f_0(980)$ and $\Upsilon'''\sigma(400-1200)$ with the QCD sum rules, and make reasonable predictions for their masses. Our predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. We can search for those bound states at the LHCb, the KEK-B or the Fermi-lab Tevatron.

## Appendix

The spectral densities at the level of the quark-gluon degrees of freedom:

$$\begin{aligned}\rho_0(s) &= \frac{3}{4096\pi^6}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\alpha\beta(1-\alpha-\beta)^2(s-\widetilde{m}_Q^2)^3(5s-\widetilde{m}_Q^2)\\&+\frac{3m_Q^2}{1024\pi^6}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta(1-\alpha-\beta)^2(s-\widetilde{m}_Q^2)^3\,, \end{aligned}\quad (9)$$

$$\begin{aligned}\rho_{\langle\bar{s}s\rangle}(s) &= \frac{9m_s\langle\bar{s}s\rangle}{128\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\alpha\beta(s-\widetilde{m}_Q^2)(3s-\widetilde{m}_Q^2)\\&+\frac{9m_sm_Q^2\langle\bar{s}s\rangle}{64\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta(s-\widetilde{m}_Q^2)-\\&\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\alpha(1-\alpha)(2s-\widetilde{\widetilde{m}}_Q^2)-\frac{m_sm_Q^2\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\,, \end{aligned}\quad (10)$$

$$\begin{aligned}\rho_{\langle\bar{s}s\rangle^2}(s) &= -\frac{\langle\bar{s}s\rangle^2}{16\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\alpha(1-\alpha)(2s-\widetilde{\widetilde{m}}_Q^2)-\frac{m_Q^2\langle\bar{s}s\rangle^2}{16\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\\&+\frac{\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\alpha(1-\alpha)\left[3+\left(3s+\frac{s^2}{M^2}\right)\delta(s-\widetilde{\widetilde{m}}_Q^2)\right]\\&+\frac{m_Q^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[1+\frac{s}{M^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2)\\&+\frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{128\pi^2M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\alpha(1-\alpha)s\left[1+\frac{s}{M^2}+\frac{s^2}{2M^4}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2)\\&+\frac{3m_Q^2\langle\bar{s}g_s\sigma Gs\rangle^2}{768\pi^2M^6}\int_{\alpha_i}^{\alpha_f}d\alpha s^2\delta(s-\widetilde{\widetilde{m}}_Q^2)\,, \end{aligned}\quad (11)$$

$$
\begin{aligned}
\rho^{A}_{\langle GG\rangle}(s) &= \frac{3}{1024\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\alpha\beta(s-\widetilde{m}_Q^2)(3s-\widetilde{m}_Q^2) \\
&-\frac{1}{2048\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta(1-\alpha-\beta)^2(s-\widetilde{m}_Q^2)(5s-3\widetilde{m}_Q^2) \\
&+\frac{3m_Q^2}{512\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta(s-\widetilde{m}_Q^2) \\
&-\frac{m_Q^2}{1024\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right](1-\alpha-\beta)^2(2s-\widetilde{m}_Q^2) \\
&+\frac{3m_Q^2}{1024\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right](1-\alpha-\beta)^2(s-\widetilde{m}_Q^2) \\
&-\frac{m_Q^4}{1024\pi^4}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right](1-\alpha-\beta)^2 \\
&-\frac{m_s\langle\bar{s}s\rangle}{128\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[3+s\delta(s-\widetilde{m}_Q^2)\right] \\
&-\frac{m_s m_Q^2\langle\bar{s}s\rangle}{128\pi^2M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right]s\delta(s-\widetilde{m}_Q^2) \\
&-\frac{m_s m_Q^4\langle\bar{s}s\rangle}{128\pi^2M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right]\delta(s-\widetilde{m}_Q^2) \\
&+\frac{3m_s m_Q^2\langle\bar{s}s\rangle}{128\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\int_{\beta_i}^{1-\alpha}d\beta\left[\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right]\delta(s-\widetilde{m}_Q^2)\,, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\rho^{B}_{\langle GG\rangle}(s) &= \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{576\pi^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[2+\frac{s}{M^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&+\frac{\langle\bar{s}s\rangle^2}{288}\int_{\alpha_i}^{\alpha_f}d\alpha\left[2+\frac{s}{M^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&-\frac{m_s m_Q^2\langle\bar{s}g_s\sigma Gs\rangle}{576\pi^2M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1-\alpha}{\alpha^2}+\frac{\alpha}{(1-\alpha)^2}\right]\left[1-\frac{s}{M^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&+\frac{m_s m_Q^4\langle\bar{s}g_s\sigma Gs\rangle}{576\pi^2M^4}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1}{\alpha^3}+\frac{1}{(1-\alpha)^3}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&-\frac{m_Q^2\langle\bar{s}s\rangle^2}{288M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1-\alpha}{\alpha^2}+\frac{\alpha}{(1-\alpha)^2}\right]\left[1-\frac{s}{M^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&+\frac{m_Q^4\langle\bar{s}s\rangle^2}{288M^4}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1}{\alpha^3}+\frac{1}{(1-\alpha)^3}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&-\frac{m_s m_Q^2\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^2M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1}{\alpha^2}+\frac{1}{(1-\alpha)^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2) \\
&-\frac{m_Q^2\langle\bar{s}s\rangle^2}{96M^2}\int_{\alpha_i}^{\alpha_f}d\alpha\left[\frac{1}{\alpha^2}+\frac{1}{(1-\alpha)^2}\right]\delta(s-\widetilde{\widetilde{m}}_Q^2)\,. \qquad (13)
\end{aligned}
$$

## Acknowledgements

This work is supported by National Natural Science Foundation of China, Grant Number 10775051, and Program for New Century Excellent Talents in University, Grant Number NCET-07-0282.

## References

[1] X. L. Wang et al, Phys. Rev. Lett. **99**, 142002 (2007).

[2] C. Z. Yuan et al, Phys. Rev. Lett. **99**, 182004 (2007).

[3] G. Pakhlova et al, Phys. Rev. Lett. **98**, 092001 (2007).

[4] G. Pakhlova et al, Phys. Rev. Lett. **100**, 062001 (2008).

[5] G. Pakhlova et al, Phys. Rev. D **77**, 011103 (2008).

[6] B. Aubert et al, arXiv:0710.1371.

[7] P. Pakhlov et al, Phys. Rev. Lett. **100**, 202001 (2008).

[8] G. J. Ding, J. J. Zhu and M. L. Yan, Phys. Rev. D **77**, 014033 (2008).

[9] B. Q. Li and K. T. Chao, arXiv:0903.5506.

[10] A. M. Badalian, B. L. G. Bakker and I. V. Danilkin, arXiv:0805.2291.

[11] C. F. Qiao, J. Phys. **G35**, 075008 (2008).

[12] R. M. Albuquerque and M. Nielsen, Nucl. Phys. A **815**, 53 (2009).

[13] R. Faccini, arXiv:0801.2679.

[14] F. K. Guo, C. Hanhart and Ulf-G. Meissner, Phys. Lett. B **665**, 26 (2008).

[15] C. Amsler et al, Phys. Lett. **B667**, 1 (2008).

[16] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B **666**, 344 (2008).

[17] M. B. Voloshin, Prog. Part. Nucl. Phys. **61** (2008) 455.

[18] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448.

[19] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.

[20] B. L. Ioffe, Prog. Part. Nucl. Phys. **56** (2006) 232.

[21] Z. G. Wang, Eur. Phys. J. **C62** (2009) 375.

[22] Z. G. Wang, Chin. Phys. **C32** (2008) 797.

[23] Z. G. Wang, Nucl. Phys. **A791** (2007) 106.

[24] Z. G. Wang, W. M. Yang and S. L. Wan, J. Phys. **G31** (2005) 971.

[25] Z. G. Wang, Eur. Phys. J. **C63** (2009) 115.

[26] M. Davier, A. Hocker, Z. Zhang, Rev. Mod. Phys. **78** (2006) 1043.

[27] F. E. Close and N. A. Tornqvist, J. Phys. **G28** (2002) R249.

[28] C. Amsler and N. A. Tornqvist, Phys. Rept. **389** (2004) 61.

[29] Z. G. Wang, W. M. Yang and S. L. Wan, Eur. Phys. J. C **37**, 223 (2004).

[30] G. Kane and A. Pierce, "Perspectives On LHC Physics", World Scientific Publishing Company, 2008.